
HHC
2025

A New Random Number Generator

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First, a bit of history

A Historical Background

- 1890: Galton's Dice
- 1946: Von Neuman's Middle Square
- 1951: Lehmer's Linear Congruential
- 1958: Lagged Fibonacci
- 1986: ACORN
- 1991: Add with Carry (Marsaglia)
- 1994: Multiply with Carry (Marsaglia)
- 2003: XOR Shift (Marsalia)

Sir Francis Galton

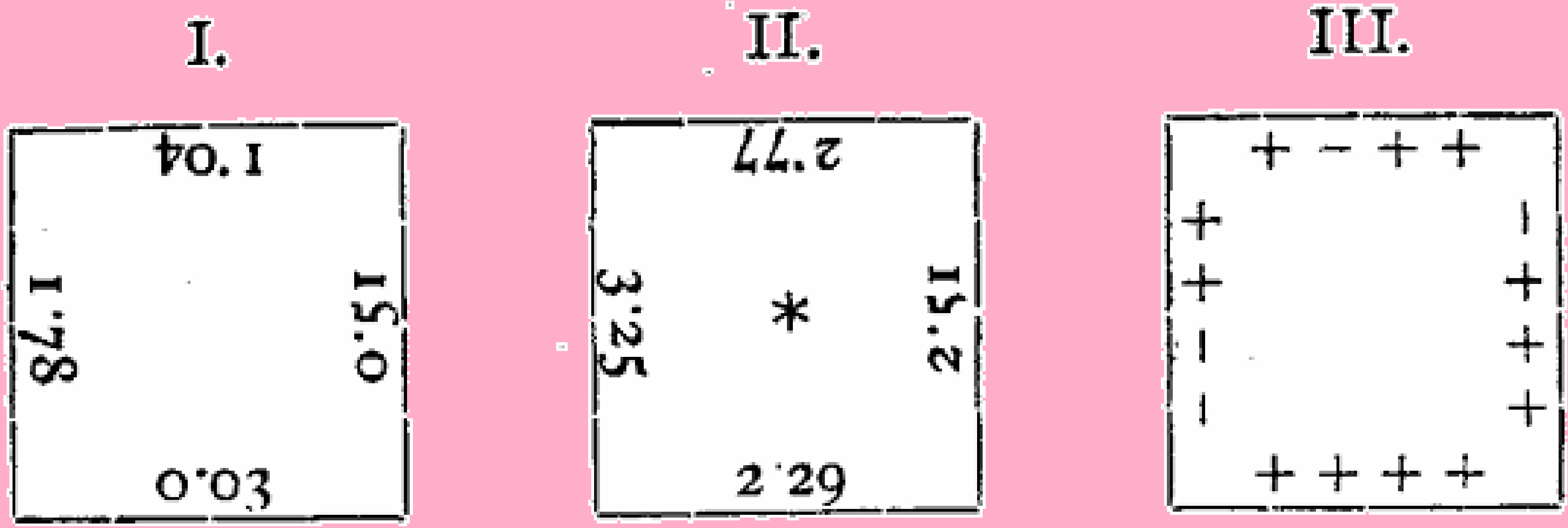
- **Francis Galton was a pioneer of modern statistics. He was among the first to use standard deviation and correlation coefficient in data analysis.**
- **Galton applied statistical methods to the study of scientific subjects, such as phrenology.**
- **Galton coined the word “eugenics”.**
- **He borrowed some ideas from his cousin, Charles Darwin.**

Galton's Random Number Generator

What's on the Die Faces?

There are three types of die

- **Type 1 for the center of the bell curve**
- **Type 2 for the tails of the curve**
- **Type 3 to assign a random sign to each throw**


I.
II.
III.
1.04
1.78
0.03
2.77
3.25
*
2.29

Galton's Numbers (Type 1 Die)

0.03	0.51	1.04	1.78
0.11	0.59	1.14	1.95
0.19	0.67	1.25	2.15
0.27	0.76	1.37	-2.4
0.35	0.85	1.5	-2.75
0.43	0.94	1.63	-3.6

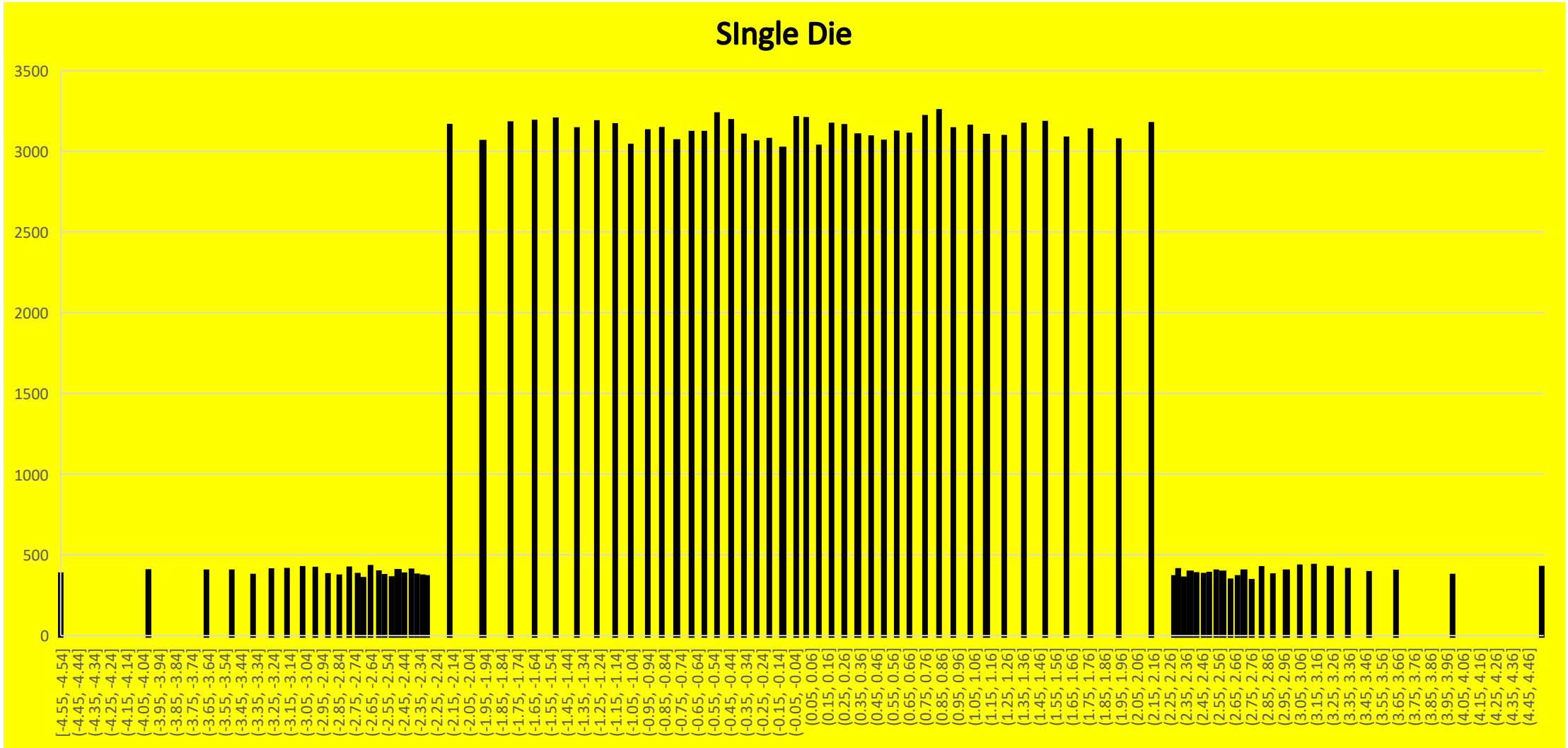
Galton's Numbers (Type 2 Die)

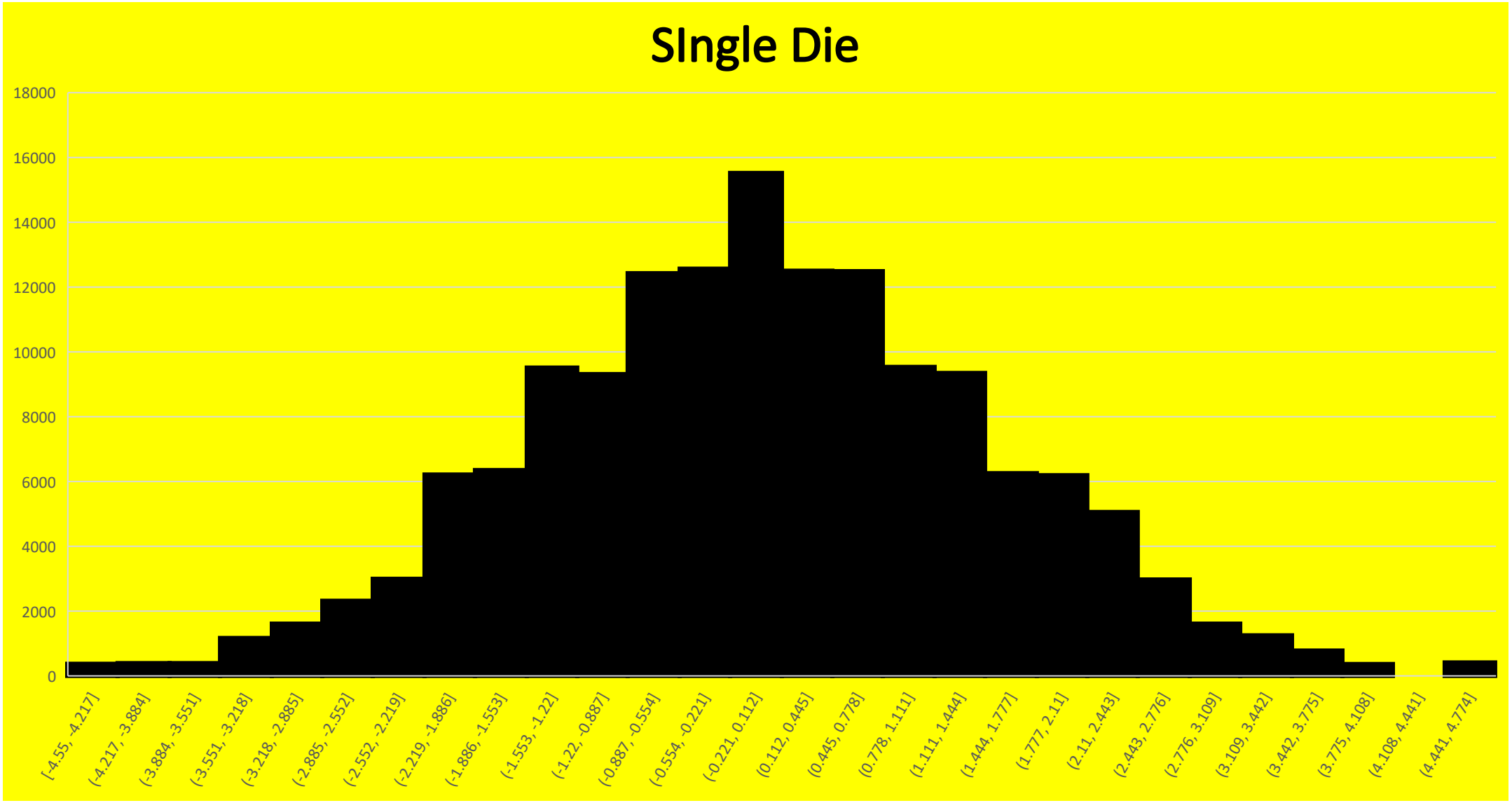
2.29	2.51	2.77	3.25
2.32	2.55	2.83	3.36
2.35	2.59	2.9	3.49
2.39	2.64	2.98	3.65
2.43	2.68	3.06	4.00
2.47	2.72	3.15	4.55

How to generate a normal deviate:

- Throw a type 1 die. Blindly orient the die so one edge faces you.
- If you get a blank or negative (three of the 24 edges are blank or negative), get your number from a type 2 die.
- apply the sign from the type 3 die to the the result.
- Galton recommends the sum of two or three throws.

Some Results 250000 Samples, more than Galton had time for




SIngle Die
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
[-4.55, -4.217]
(-4.217, -3.884]
(-3.884, -3.551]
(-3.551, -3.218]
(-3.218, -2.885]
(-2.885, -2.552]
(-2.552, -2.219]
(-2.219, -1.886]
(-1.886, -1.553]
(-1.553, -1.22]
(-1.22, -0.887]
(-0.887, -0.554]
(-0.554, -0.221]
(-0.221, 0.112]
(0.112, 0.445]
(0.445, 0.778]
(0.778, 1.111]
(1.111, 1.444]
(1.444, 1.777]
(1.777, 2.11]
(2.11, 2.443]
(2.443, 2.776]
(2.776, 3.109]
(3.109, 3.442]
(3.442, 3.775]
(3.775, 4.108]
(4.108, 4.441]
(4.441, 4.774]

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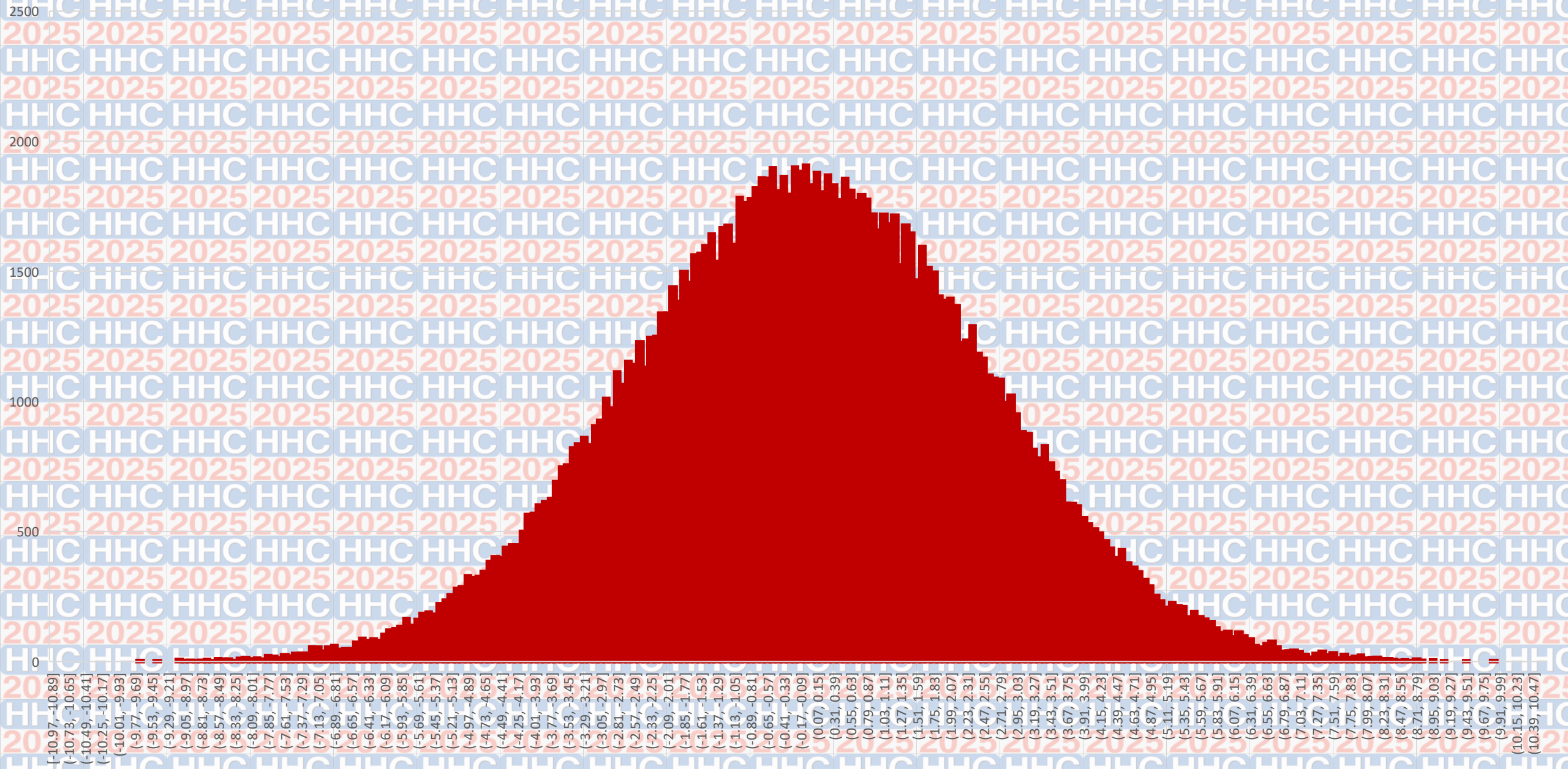
Three Dice



Some Stats…

- All 576 equally likely combinations were calculated. Mean was zero (by symmetry) and standard deviation about 1.477562.
- Skew, serial correlation, chi square bins, comb tests, etc… not tested.

Conclusions

- Histograms look “normal” to me…Pretty good for 1890. but in these times we demand better (and faster).
- Two faults: granularity and weak tails.
- Central limit theorem buries a lot of blemishes.
- This technology could be applied to other distributions.
- Much easier than counting heads in 40000 coin flips, subtracting 20000, and dividing by 100.

Galton's Logic:

- For type 1 dice, Galton seems to have divided the cumulative probability range [.5, 1.0] into 24 equal intervals of probabability 1/48. For type 2 dice, the range [.9375, 1.000] was divided into 24 intervals of probability 1/384.
- The probability integral transform is applied to each boundary value and a number is selected from each of the transformed intervals.
- The method of choosing a sample point within each interval is not clear. However, the RMS value for that interval has the same contribution to variance as the entire interval. The median point is close to that and much easier to calculate.
- Galton's standard deviation is 1.477562, which may have been intended for a specific problem. (I calculated this from RMS of all 576 equally likely outcomes in the sample space).

Why Cubical Dice?

- Galton was no idiot; cubical dice are easy to make from stock lumber.
- Octahederal dice no better; only 24 face edges.
- Suggestions on how to make dodecahederal dice will be welcome.

Possible Improvements

- All numbers on dice to have three decimal places.
- Help the tails by adding a third die (that would rarely be used).
- Help the granularity by having the standard deviation be 0.7071 and rolling twice. The second number would be added or subtracted, according to coin flip.

Improvements?

- More digits? Would be a big help.
- More rolls? Better quality, but slower and greater error probability
- Dice with more edges? some possibilities…

https://commons.wikimedia.org/w/index.php?curid=87223450

Extending Galtons Idea

Type 1 die: The numbers on the edges are: *

0.015	0.046	0.913	0.970
0.142	0.176	0.702	0.752
0.281	0.318	0.519	0.563
0.435	0.476	0.356	0.395
0.608	0.654	0.210	0.245
0.804	0.857	0.077	0.109

* highs and lows on same face to mitigate die unbalance

Richard's Type 2 Die

- Three faces marked "0"
- Three faces marked "1"
- Coin flips may be used in lieu of die 2

How to use exponential distribution dice

- Throw a type 1 die. Blindly orient the die so one edge faces you. That is your total.
- Throw type 2 die (or flip a coin). If 0, you are done. If you get a 1, add it to the total. (Or flip a coin with head 1 and tail 0).
- Repeat step 2 above. (There is a very small probability that the sun will go out before you get a 0.)
- What you get is an exponential distribution with median 1 and both mean and variance 1/Ln(2).

Last Slide: A Fun Problem

How could the exponential distribution dice be used to draw from a Poisson distribution?

Reference:

Galton, Sir Francis, “Dice for Statistical Experiments”, Nature 1070 Vol. 42, May 1 1890, p. 13.